Une approche basée sur SAT

pour le problème de satisfiabilité en logique modale S5

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Preliminaries: Modal Logic

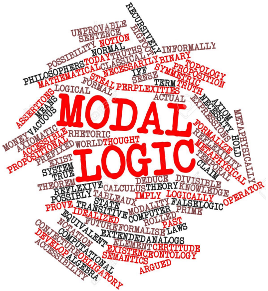
Modal Logic = Propositional Logic + $\Box$ and $\Diamond$

Modal Logic

- $\Box\phi$ means ϕ is necessarily true
- $\Diamond\phi$ means ϕ is possibly true

$$\Diamond\phi \leftrightarrow \neg\Box\neg\phi$$

$$\Box\phi \leftrightarrow \neg\Diamond\neg\phi$$



Sum-up of complexities

NP
K5
K45
KB45
KD5
KD45
S5

PSPACE
K
KT
S4
KB
KD4
KD
K4
KDB
KBT

Proofs of complexities are in [Lad77, HR07]

Preliminaries: S5-Structure

- $\mathbb{P}$ finite non-empty set of propositional variables

S5-Structure [Kri59]

$M = \langle W, R, V \rangle$ with:

- W, a non-empty set of possible worlds
- R, a binary relation on W (which is total: $\forall.w\forall.v\ (w, v) \in R$)
- V, a function that associate to each $p \in \mathbb{P}$, the set of possible worlds where p is true

Pointed S5 Structure: $\langle \mathcal{K}, w \rangle$

- $\mathcal{K}$: S5 Structure
- w is a possible world in W

Preliminaries: Satisfaction Relation

Definition (Satisfaction Relation)

The relation $\models$ between S5 Structures and formulae is recursively defined as follows:

$$
\begin{array}{lll}
\langle \mathcal{K}, w \rangle \models p & \text{iff} & w \in V(p) \\
\langle \mathcal{K}, w \rangle \models \neg\phi & \text{iff} & \langle \mathcal{K}, w \rangle \not\models \phi \\
\langle \mathcal{K}, w \rangle \models \phi_1 \wedge \phi_2 & \text{iff} & \langle \mathcal{K}, w \rangle \models \phi_1 \text{ and } \langle \mathcal{K}, w \rangle \models \phi_2 \\
\langle \mathcal{K}, w \rangle \models \phi_1 \vee \phi_2 & \text{iff} & \langle \mathcal{K}, w \rangle \models \phi_1 \text{ or } \langle \mathcal{K}, w \rangle \models \phi_2 \\
\langle \mathcal{K}, w \rangle \models \Box\phi & \text{iff} & \forall v \in R \text{ we have } \langle \mathcal{K}, v \rangle \models \phi \\
\langle \mathcal{K}, w \rangle \models \Diamond\phi & \text{iff} & \exists v \in R \text{ such that } \langle \mathcal{K}, v \rangle \models \phi
\end{array}
$$

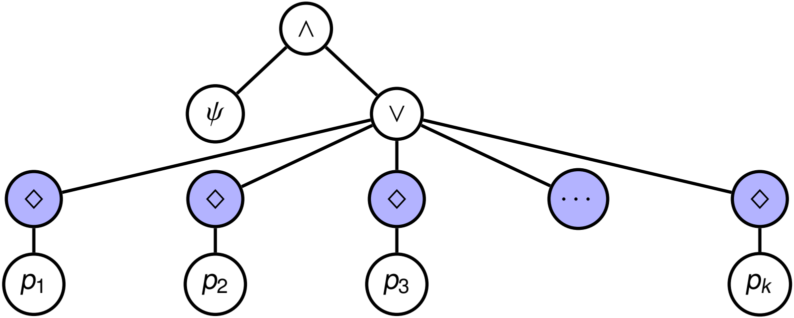
$\mathcal{K}$ that satisfied a formula ϕ will be called “model of ϕ”

Diamond-Degree: Strictly better than nm

Comparison dd(ϕ) vs nm(ϕ)

$\text{nm}(\phi) = \text{nm}'(\text{nnf}(\phi))$	$\text{dd}(\phi) = \text{dd}'(\text{nnf}(\phi))$
$\text{nm}'(p) = \text{nm}'(\neg p) = 0$	$\text{dd}'(p) = \text{dd}'(\neg p) = 0$
$\text{nm}'(\phi \wedge \psi) = \text{nm}'(\phi) + \text{nm}'(\psi)$	$\text{dd}'(\phi \wedge \psi) = \text{dd}'(\phi) + \text{dd}'(\psi)$
$\text{nm}'(\phi \vee \psi) = \text{nm}'(\phi) + \text{nm}'(\psi)$	$\text{dd}'(\phi \vee \psi) = \max(\text{dd}'(\phi), \text{dd}'(\psi))$
$\text{nm}'(\Box\phi) = 1 + \text{nm}'(\phi)$	$\text{dd}'(\Box\phi) = \text{dd}'(\phi)$
$\text{nm}'(\Diamond\phi) = 1 + \text{nm}'(\phi)$	$\text{dd}'(\Diamond\phi) = 1 + \text{dd}'(\phi)$

Diamond-Degree: Strictly better than nm



- nm(φ) equals k
- dd(φ) equals 1

- We just need to satisfy 'one' diamond, not all of them
- The entire formula needs only dd(φ) $+$ 1 worlds

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Modal Logic S5 solver: S52SAT

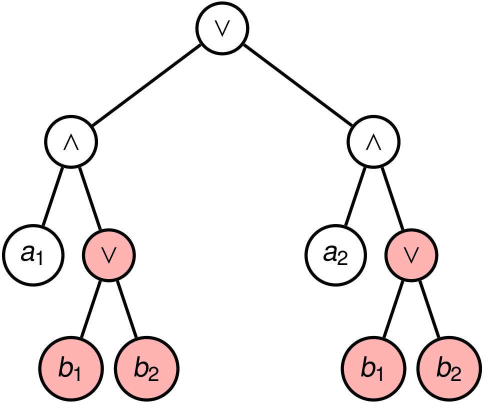
- Translation from S5-SAT to SAT.
- Polynomial reduction: S5-SAT is NP-complete [Lad77]

Translation from S5 to SAT

$$\mathrm{tr}(\phi, n) = \mathrm{tr}(\mathrm{nnf}(\phi), 1, n)$$
$$\mathrm{tr}(p, i, n) = p_i$$
$$\mathrm{tr}(\neg p, i, n) = \neg p_i$$
$$\mathrm{tr}(\Box\phi, i, n) = \bigwedge_{j=1}^{n}(\mathrm{tr}(\phi, j, n))$$
$$\mathrm{tr}(\Diamond\phi, i, n) = \bigvee_{j=1}^{n}(\mathrm{tr}(\phi, j, n))$$

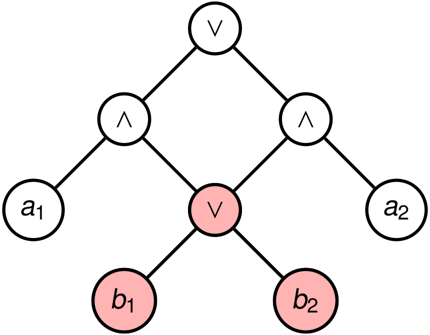
Modal Logic S5 solver: S52SAT

Let $\phi = \Diamond(a \wedge \Diamond b)$ as example (with n = 2).



Modal Logic S5 solver: S52SAT - structural caching

$\phi = \Diamond(a \wedge \Diamond b)$, with caching.



Modal Logic S5 solver: S52SAT - with/without caching

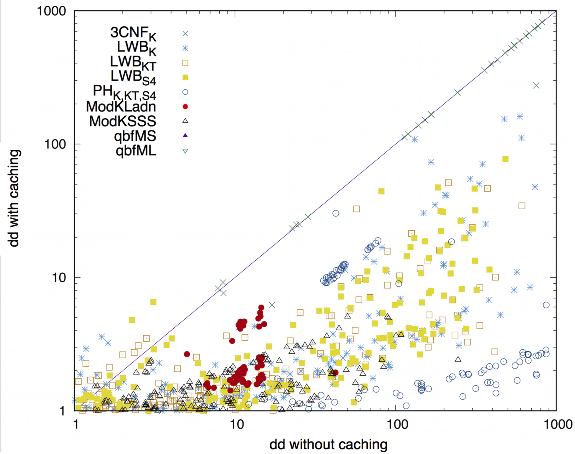


Figure: Scatter plot with/without caching

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Modal Logic S5 solver: S52SAT - against SotA solvers

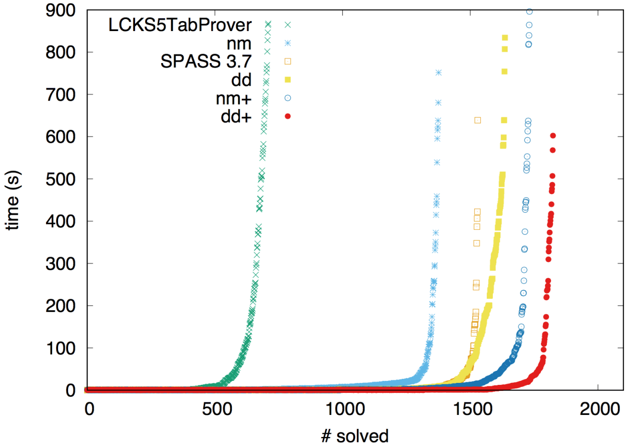

Figure: Cactus-Plot of the runtime distributions

Conclusion

Conclusion

- S52SAT the most efficient approach on benchmarks considered
- Benchmarks considered are not “real” problems
- Modal Logic S5 is more expressive than SAT
- S52SAT returns S5-model when it found one

Perspective

- Adapting S52SAT to solve other modal logics in *NP*
- Returning the smallest S5-model possible

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